Mathematical Modeling as a tool for Basic Research in Acupuncture

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Abstract: We propose, and illustrate a five step strategy for basic research of traditional explanatory frameworks of acupuncture. Our approach is based on utilizing mathematical models as a bridge between the traditional explanatory frameworks of acupuncture and Western research technologies.
Step 1. Carefully select and document case studies which would allow simultaneous interpretation within several "traditional" explanatory frameworks of acupuncture.
Step 2. Develop minimal theoretical models connecting the diagnosis and treatment within the context of each of the selected explanatory frameworks of acupuncture.
Step 3. Develop minimal dynamical systems models for each theoretical model from Step 2, so as to tighten their logical structure and to bring them into a falsifiable and more abstracted format (which provides the link between the theoretical models at Step 2 and possible western based models, and hence western measurement technology).
Step 4. Interpret the mathematical models from Step 3 within the framework of "Western" scientific perspectives.
Step 5. Select and conduct appropriate "objective" (skin electrica impedance, etc) measurements to test the validity of the models at Step 4.
Iterate Steps 1 - 5 as needed.

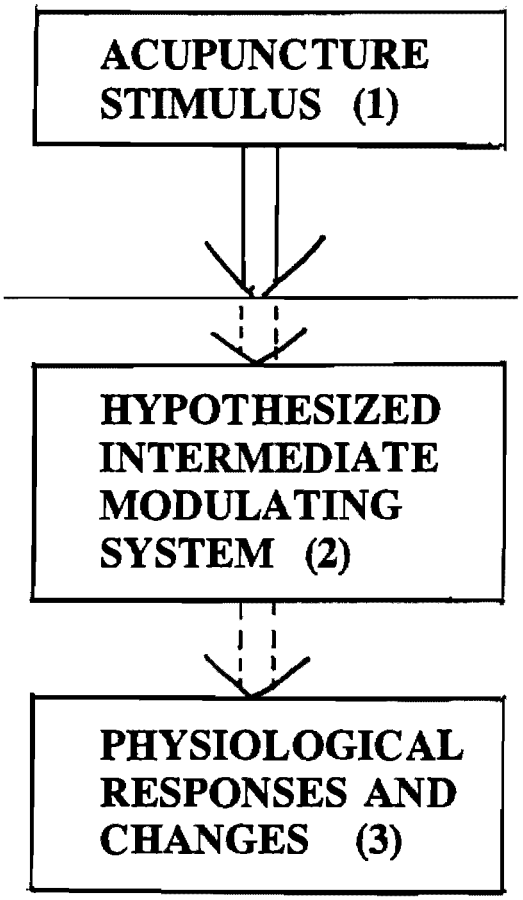
0. Introduction

Acupuncture, the use of needles and related ancillary techniques to regulate physiology and relieve symptoms has been practiced in China for over 2000 years, and in Japan and Korea over 1450 years. It arrived in Europe around 300 years ago and North America over 150 years ago [Birch & Felt 1997, Needham 1980], recently spreading to most countries around the world [Cheng 1987, W.H.O. 1985]. The earliest major documents on acupuncture are revered as the primary sources of acupuncture and have been commented on by numerous authors over the centuries [Sivin 1987, Ishihara 1983, Unschuld 1986]. For many acupuncturists today, these texts are still highly regarded, studied, and provide the basis for much of modern practice. Frequently the authenticity of a particular tradition or practitioner's approach is claimed by citing appropriate passages from these ancient and later texts [Connelly 1979, Kaptchuk 1983, Maciocia 1989, Matsumoto & Birch 1988, Shudo 1990, Soulie de Morant 1994]. Differences within [Epler 1980] and between [Birch 1992] these texts and socio-political and cultural variations in each country where acupuncture has been practiced, have fostered a wide variety of conceptual models, methods and techniques of practice [Birch & Felt 1997, Needham 1980]. The use of a single term 'acupuncture' can be misleading as it implies a coherent and uniform model of practice, which is not the case. Not only is there historical and modern evidence of the use of a wide range of treatment techniques [Birch 1997a] and diagnostic assessment methods [Birch & Felt 1997, Birch 1992, Birch 1997a], but the conceptual frameworks by which acupuncture has been practiced also show considerable variety, both in traditionally based

and modern frameworks. There were many historical systems [Birch & Felt 1997, Sivin 1987, Unschuld 1986, Matsumoto & Birch 1988, Matsumoto & Birch 1986], and today there are many *traditionally based systems of acupuncture* (TBSAs), such as traditional Chinese medical (TCM) acupuncture [Cheng 1987, Kaptchuk 1983, Maciocia 1989], Traditional or Five-Element Acupuncture [Connelly 1979], pre-TCM acupuncture [Soulie de Morant 1994, So 1987], Meridian Therapy [Shudo 1990], Yin-yang channel balancing therapy, [Manaka, Itaya, Birch 1995] Medical Acupuncture, [Helms 1995] and many modern systems of practice [Baldry 1989, Nakatani & Yamashita 1977, Ulett 1992].

Each TBSA has its own unique methods of assessing patients, deciding what the problem is, and deciding what acupuncture points and techniques to use. As we will see below, were the same patient to be assessed by practitioners of different TBSAs, the labels and treatments would all be different [Birch &Felt 1997, Birch 1995]. What the systems all have in common is the idea that the treatment of specific points using specific techniques for each individual patient can maximize the efficacy of treatment. This is interesting because, in each TBSA, so long as a diagnosis is arrived at, a treatment automatically follows [Birch &Felt 1997, Birch 1997b]. It thus appears that the traditional concepts upon which each system is based, and their associated assessment methods ultimately guide the determination of what points to treat with what techniques. This leaves the researcher interested in testing acupuncture with many questions about the nature of these TBSAs. Since each traditional framework and its associated methods of practice guide the practitioner in their selection of treatment points and associated techniques for each patient, then either:
1) the explanatory frameworks are nothing but heuristic devices or techniques that give the practitioner guidelines for selecting treatment points and techniques, in which case all the traditional concepts - qi, jing luo, zang fu, yin yang etc, may have no basis in physical measurable reality, or
2) the explanatory frameworks do refer to some phenomena in nature, which the various assessment and treatment methods take advantage of to treat patients, in which case the traditional concepts may have an as yet untested, unmeasured basis in physical reality.

According to practitioners of TBSA approaches, the individualization of treatment based on traditional concepts and practices produces greater treatment effects than when acupuncture is used without the use of these methods. However, since no clinical trials of acupuncture have been conducted that compared the various forms of acupuncture to each other, or have even tested TBSA approaches, [Birch 1997b] there is no basis for judging if this claim is correct. Further, since no studies have been adequately designed that could systematically explore and test these traditional frameworks, their concepts and clinical methods, it is appropriate to begin designing studies that allow us to test such claims. Dismissing these claims as nonsense [Ulett 1995] may reflect cultural bias and an inadequate grasp of the subject matter, more than it does good scientific judgement. On the other hand, insisting that the paradigm of western (reductionist) research cannot be used to test the "holistic" paradigm of acupuncture [Patel 1987], is not only a gross simplification of the nature of acupuncture [Unschuld 1987, Unschuld 1992], but the argument makes faulty assumptions about the nature of the so-called "paradigms" [Vickers 1996, Birch 1997a]. Appropriate studies have not yet been designed that allow us to make a scientific judgement one way or the other [Birch 1997b]. Essentially the basis of TBSAs can be summarized in the following diagram:



(1) Acupuncture is a complex diverse system of medical interventions with multiple stimulus inputs.

Boundary of Body (skin)

(3) Observed physiological events eg:
- neuropeptide and concurrent physiological changes;
- circulatory and concurrent physiological changes.

(2) Traditionally a complex system of points (jing xue), channels (jing luo), organs (zang fu) each composed of and influencing an elaborate construct (the qi system), regulated by a series of operating principles (yin-yang, wu xing [five phases]) was said to mediate the effects of the acupuncture stimulus inputs. It was hypothesized that if this system was taken advantage of (by guiding the assessment of patients and selecting stimulus inputs) then treatment effects could be maximized.

DIAGRAM

(Diagram here)

In order to test this hypothesized intermediate modulating system, the authors propose a step-by step approach:
Step 1: carefully select and document case studies which would allow simultaneous interpretations within several TBSA approaches;
Step 2: develop minimal theoretical models connecting the diagnosis and treatment within the context of each of the selected TBSA frameworks;
Step 3: develop minimal dynamical systems models for each theoretical model from Step 2, so as to tighten their logical structure and to bring them into a falsifiable and more abstracted format (which provides the link between the theoretical models at Step 2 and possible western based models, and hence western measurement technology);
Step 4: interpret the mathematical models from Step 3 within the framework of "western" scientific perspectives;
Step 5: select and conduct appropriate "objective" (skin electrical impedance, etc.) measurements to test the validity of the models at Step 4;
Iterate Steps 1-5 as needed.

The remainder of the paper is organized as follows. In Section 1 we present a model case study and the treatments according to three TBSAs. In Section 2 we develop a simple (mathematical) model of disease which provides us with a coherent framework for implementing Step 2. In Section 3 we address Step 3. See [Birch & Friedman 1989, Friedman, Birch, Tiller 1989a, Friedman, Birch, Tiller 1989b, Friedman & Birch 1994] for our earlier work related to steps 2, 3. In Section 4 (Steps 4, 5), we outline an approach which links the TBSA mathematical models from Step 3 with electrodermal measurements. Section 5 contains conclusions.

1. A model case study; 3 treatments.

A model case study: Examination.
Female age 37. Main complaints: cystitis, dysmenorrhea, and chronic low back pain.
Visual inspection: yellowish around the eyes; thin tight muscled frame; thin, red tongue body with yellow fur at the rear of the tongue; presence of pigmentation in the interscapular region with moles on the sides of the neck and abdomen; vascular spiders visible on the costal region over the liver.
Olfactory and auditory inspection: a slightly-sickly sweet body odor; a singing-like, lilting tone of voice.
Questioning: dislikes hot weather; the low back pain is better with pressure applied to the back; hands generally feel very warm; feels tired and hot in the late afternoon; is generally very pensive and easily irritated; the urine flow is urgent, painful, obstructed and quite yellow in color; the menstrual flow is heavy with some pain and clotting; sometimes has a problem with sweating at night.
Palpation: the second right deep pulse is the weakest, the second and third left deep

pulses are also quite weak; the pulse is overall deep, thin and fast; the abdomen exhibits stiffness in the right subcostal region and above the navel, with pressure pain and stiffness in the areas immediately to either side of the navel and in the area about two inches to the left of the navel, extending below the level of the navel.

As Step 1, the following table summarizes the (i) TCM acupuncture [Cheng 1987, Kaptchuk 1983, Maciocia 1989]; (ii) meridian therapy [Shudo 1990]; and (iii) yin-yang channel balancing treatments [Matsumoto & Birch 1988, Manaka, Itaya, Birch 1995] that would be given to the same patient, described above, were she assessed and treated by practitioners of each TBSA. This case is described in detail elsewhere, [Birch & Felt 1997, Birch 1995] but sufficiently illustrates 3 TBSA approaches that will be further explored in Steps 2–4.

TBSA	*points*	*technique*	*depth*	*SUBJECTIVE SENSATION*
(i)	KI-3, BL-52 CV-4 , SP-9, BL-28, BL-40	Supplement Drain	7.5-37.5 mm 12.5-37.5 mm	de qi de qi
(ii)	LV-8, KI-10 CV-4, BL-18 BL-23 SP-3	Supplement Supplement Supplement Drain	1-2 mm 1-4 mm 1-4 mm 1-2 mm	none none none none
(iii)	PC-6, SP-4 TB-5, GB-41 BL-18,BL-23 BL-28	IP cord IP cord needle-moxa needle-moxa	2 mm 2mm 10-15mm 15mm	none none heat heat

2. Theoretical rationale for the prescribed treatments. A simplified mathematical model of disease

Each treatment framework has a somewhat different rationale for how to assess and treat the patient. In these examples, each is based at least in part on some of the traditional concepts and methods that date back some 2,000 years. In order to gain a clear overview of each treatment framework, it is necessary to grasp the basic components of the TBSAs themselves.

Essentially, the historical explanations of human physiology, pathology, diagnosis and treatment are based on the concept of "qi", which is variously translated as "influences", "finest matter influences" and more popularly and less accurately rendered as "energy" or "vital energy". In relation to life and physiology, this "qi" is said to take many forms, have many functions, circulates in the body, is derived from a reservoir given at birth, and from assimilation from the environment after birth through breathing and digestion. In some representations of this concept, the "qi" is as a seamless whole, where all of its aspects at least within the body are affected by and affect all others. The "qi" circulates along pathways, primarily the "jing mai" or channels", with a variety of secondary "channel" systems. It is produced in the processes of assimilation, and transformation of stored aspects by the "zang-fu" or "organs". The "jing mai" and "zang-fu" are interconnected in an intricate system of processes and relationships. General theoretical constructs such as

"yin yang" and the "wu xing" or "five phases" are described with reference to regulatory principles that govern all aspects of the "qi" system, and also label-categorize all of those aspects and their derivative structures as the organizing principle or skeleton onto which the body of "qi" systems is hung. "Qi" is expended throughout life by normal physiology and activities and in response to disease processes. Disease processes are often labelled with reference to which aspects of the overall structure are awry, whether it is not circulating correctly, whether it is not being produced correctly, whether too much of it is being expended fighting pathogens and disease processes.

Here is the simplest description of the condition and content of "qi" in the body:

$$Q = Q_p + Q_e + Q_{cd}.$$

Here Q = total amount qi in the person or an organ/functional unit (+ related substances such as blood), Q_p = production of qi (respiration, digestion, assimilation, conversion of reserves, transformation of stored substances) Q_e = expenditure of qi (activities, excretion, and normal physiology + fighting disease [pathogenic qi]) Q_{cd} = circulation and distribution of qi (the whole channel system, both primary and secondary systems).

Different TBSAs draw upon different components of this simple equation, and, assuming the seamless interconnectedness of the whole system of "qi", each assumes an impact upon each of the unstated components. For example, it is possible to view ***(1)*** the "ba gang bian zheng" or traditional Chinese medical acupuncture diagnosis and treatment simply in terms of weakness of qi production Q_p, (kidney yin vacuity) and qi expenditure, Q_e to process, eliminate the pathological factors (damp heat in lower jiao); the circulation and distribution of qi Q_{cd} is assumed to be adjusted once these other factors are corrected. In the same vein, ***(2)*** the "keiraku chiryo" or Japanese channel therapy acupuncture diagnosis and treatment can be viewed in terms of a problem of the circulation and distribution of qi, Q_{cd} (liver vacuity and spleen repletion); the production (Q_p) and expenditure (Q_e) of qi are assumed to be adjusted once these circulation problems are corrected. ***(3)*** The "yin yang channel balancing therapy" acupuncture diagnosis and treatment can also be viewed in terms of the circulation and distribution of qi, Q_{cd} ("cross syndrome", liver and blood stasis); the production (Q_p) and expenditure (Q_e) of qi are also assumed to be adjusted once the circulation problem is corrected.

This simplified view of the complex events and phenomena that have been historically described in relation to health, physiology and disease can be used to picture the diverse frameworks historically described and those used in modern practice.

A further issue of importance in grasping these diverse frameworks of diagnosis and treatment is that of explaining how the treatments work (that is in terms of these historical concepts). Again, we have a simplified model that we believe can capture the salient points of each framework.

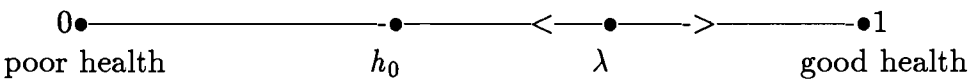
In the interpretation of a typical practitioner, the traditional frameworks have as a basic assumption, the concept of health as a "balance" as it were, of all the diverse components that comprise the traditionally formulated frameworks. Since all of these components have yin-yang and five phase correlates, we find traditional formulations of health in terms of a kind of "yin-yang balance" and "five phase balance". Disease on the other hand is seen as a process where "unbalanced" conditions have developed. In a healthy "balanced" condition, disease causing factors can be resisted and overcome. In a diseased "unbalanced" condition,

these disease factors are more likely to create further disease, making the condition more "unbalanced".

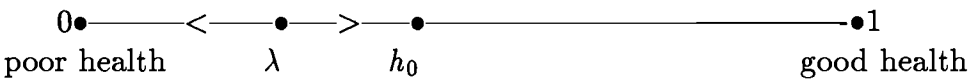
We believe that the terms "balance" and "imbalance" are modern simplifications of the TBSAs . We use instead, what we believe to be, a more precise interpretation of health and disease in terms of stable states. When an individual is *healthy* they are in a stable state by which we mean that the relevant physiological parameters are in the normal range and return to this normal range if slightly perturbed. When an individual is *diseased* they are in another stable state where relevant physiological parameters are in the abnormal range and return to this abnormal range if slightly perturbed. We recognize that the abnormal physiological parameters of the diseased state are assessed by practitioners of the various TBSAs. In some, such as the ba gang bian zheng, the pathological labels primarily focus on an alternate model of physiology and its "imbalance", in others, such as the keiraku chiryo system, the pathological labels primarily refer to conditions that are out of "balance" causing a breakdown of the normal regulation of physiology. Practitioners of these traditional frameworks of practice use the looser terms of "yin yang balance" or "five phase balance" to refer to their assessments of the patient, without clearly stating what physiological parameters are included in this. As was suggested earlier, these traditional frameworks may only be heuristic devices that organize treatment point selection. But this is the question to be investigated.

Traditionally and in modern practice, health and the treatment of disease are formulated both in terms of these global body perspectives (qi, yin yang, five phases) and the more localized disease symptoms themselves. Two basic approaches to treatment are typically utilized, the "zhibenfa", targeting the overall health of the patient, expressed in terms of qi, yin yang, the five phases etc, and the "zhibiaofa" targeting the relief of specific symptoms. In essence the *health* of a patient refers to a combination of two factors:
1) The overall health state of the patient as judged by the 4 diagnostic inspections as used in the case above. We lump them together and denote this combined *health-state* variable by h. We further make a convention that when these various categories lumped together are overall close to normal, we say that h is high, and h is low when they are far from normal.
2) We next introduce a parameter λ (lambda), which accounts for the patients' ability to recover from disease and resist it. If λ is high, this ability is poor, if λ is low, this ability is good. We represent this diagrammatically as follows:

CASE 1



CASE 2



Here the "health region" is between λ and 1; it is small in CASE 1, and it is large in CASE 2. The "disease region" is between 0 and λ; it is large in CASE 1, and it is small in CASE 2. In each region the arrow points to the direction of change of health in time: when the initial symptoms h_0 are in the "disease region", they will become worse

with time (the arrow points to the left); when the initial symptoms h_0 are in the "health region", they will improve with time (the arrow points to the right). As another way of understanding this, imagine that the line between 0 (poor health) and 1 (good health) is a cross section through a hill where λ represents the highest point on the hill, and 1 and 0 are at the same level. If a ball is placed to the left of λ it will naturally roll down towards 0, likewise if the ball were placed to the right of λ it would roll down towards 1.

With the same initial symptoms, h_0, in both cases, in CASE 1 (large "disease region") the disease will progress, while in CASE 2 (small "disease region") the patients' health will improve with time. The patient in case 1 will generally be unhealthy, typically with many minor health complaints, none of which progress rapidly to a serious stage, but generally their condition will progress. The patient in case 2 will generally be very healthy, and when symptoms do develop they are quickly resolved.

The above intuitive description of "health dynamics" can be easily formulated in a more precise form using the (mathematical) language of dynamical systems as follows.

We assume that the organism can be subdivided into separate (functional) units. We let $h(t)$ be a (generic) state variable (=lumped together symptoms) which accounts for the health state of a functional unit. Let $Q(t)$ be the principle state variable which refers to the amount of qi in a particular unit as a whole (below, we will use subscripts, when we want to specify these units or types of qi). We are interested in describing effects of $Q(t)$ on the evolution of $h(t)$ in time t.

For simplicity, we assume that a functional unit can be in either of the two homeostatic states: a diseased one or a healthy one (this implies that the organism 'adapts' to a diseased condition, which is often observed in practice). Mathematically we describe these as two stable steady states $h(t) = 0$ and $h(t) = 1$ of the function $h(t)$. Then the process of change of health in time is described by the following differential equations:

$$(1) \qquad \begin{aligned} &\frac{dh}{dt} = h(\lambda - h)(1 - h), \quad h(0) = h_0, \quad 0 < t, \\ &\frac{d\lambda}{dt} = f(Q), \quad 0 < t. \end{aligned}$$

The first equation describes the dynamics of $h(t)$. It is easy to show that $h = 0$ and $h = 1$ are the two stable steady states. In particular, the value of parameter λ determines whether the functional unit becomes "more healthy" or "more diseased" with time and how fast. Specifically, if $h_0 < \lambda$, then the initial state of the patient is in the "disease region", which means that it deteriorates with time: $h(t) \to 0$, $as\ t \to \infty$; while if $h_0 > \lambda$, then the initial state of the patient is in the "health region" which means that it improves with time: $h(t) \to 1$, $as\ t \to \infty$. The second equation accounts for the effect of qi on the "health dynamics". If $f(Q) > 0$ then $\lambda(t)$ increases with time, decreasing the "health region", while $f(Q) < 0$ would lead to the increase of the "health region".

A typical healing process can be described as follows. Suppose that initially $0 < h(0) < \lambda(0)$, i.e. $h(t)$ is in the "disease region". Then after the treatment (or a series of treatments) at the time t_1 $\lambda(t) - h(t)$ crosses zero, and from now on $0 < \lambda(t) < h(t) < 1$: $h(t)$ is in the "health region", i.e. the patient's health starts to improve, and this process continues.

We saw that in acupuncture, there are 2 basic treatment approaches: treating to correct the perceived problems and their causes (qi system), and treating to relieve symptoms. In simple terms, qi based treatments target increasing the "health region" (lowering λ), while symptom relief treatments target improving health in the sense of relieving the symptoms

(increasing the "health state" variable $h(t)$). Our simple mathematical model accounts for a variety of clinically observed phenomena. For example, consider a patient, whose symptoms improved after a treatment, but after a while worsened again. Explanation: Initially, $h(0) < \lambda(0)$, where $\lambda(0)$ is large enough. After treatment, though $h(t) > h(0)$, but λ did not significantly change, and we still have $h(t) < \lambda$. Hence the patient's state will start deteriorating again.

Note, that this simple mathematical model of health and healing capabilities does not depend on particular treatment methods or conceptual and clinical frameworks. It can therefore be used as a tool to link the diverse treatment approaches in the different TBSAs, thus providing a unified perspective to account for some essential elements in these TBSAs.

3. Some minimal dynamical systems models for the model case study

1. Ba gang bian zheng - traditional Chinese medical acupuncture. A simplified interpretation of the diagnosis: 'kidney yin vacuity' means that the production rate, call it q_p of qi is too low; 'damp heat in the lower jiao' is a disease, that requires increase expenditure rate of qi, call it q_e.

A mathematical model for the treatment is as follows. Let t_0 be the time duration of the treatment, and T be the time, during which the effects of the treatment last. We can assume that t_0 is much less than T ($t_0 \ll T$). Assume also that all quantities in the equations below are positive.

$$
\begin{aligned}
&a)\ \frac{dq_p}{dt} = b_p, \quad q_p(0) = q_{p0}, \quad 0 < t < t_0,\\
&b)\ \frac{dQ_p}{dt} = a_p q_p, \quad Q_p(0) = Q_{p0}, \quad 0 < t < T,\\
&c)\ \frac{dq_e}{dt} = b_e, \quad q_e(0) = 0, \quad 0 < t < t_0,\\
&d)\ \frac{dQ_e}{dt} = a_e q_e, \quad Q_e(0) = Q_{e0}, \quad 0 < t < T,\\
&e)\ Q(t) = Q_p(t) - Q_e(t), \quad 0 < t < T.
\end{aligned}
\tag{2}
$$

The first equation describes increasing, during the time interval $[0, t_0]$ of the treatment, the production rate of qi due to supplementing KI-3, BL-52, CV-4. The second equation describes the process of production of qi. The third equation describes turning on, during the time interval $[0, t_0]$ of the treatment, the process of expenditure of qi (to counteract the disease) due to draining SP-9, BL-28, BL-40. The fourth equation describes the process of expenditure of qi. The fifth equation is the balance equation of qi.

The healing process is described by the differential equation:

$$
\frac{d\lambda}{dt} = -Q_e, \quad 0 < t < T. \tag{3}
$$

Since $\frac{d\lambda}{dt} < 0$, qi is spent for increasing the size of the "health region", see eq. (1).

2. Keiraku Chiryo - Channel Therapy. In this case the circulation – distribution Q_{cd} system is problematic. A simplified diagnosis: 'liver and kidney vacuity', which means that the levels Q_c and Q_m of the liver (child) and kidney (mother) qi, respectively, are too low.

A mathematical model for the treatment is as follows. As before, we assume that $t_0 \ll T$, and that all quantities in the equations below are positive. A treatment taking advantage of the five phase interactions in the case of the above channel vacuity would be that of "supplementing" LV–8 and KI–10. This can be expressed as follows:

(4)

$$a)\ \frac{dq_{m,c}}{dt} = b_{m,c}, \quad q_{m,c}(0) = 0, \quad 0 < t < t_0,$$

$$b)\ \frac{dQ_c}{dt} = -a_c(Q_c - Q_c^0) + a_{m,c}(Q_m - Q_m^0) + a'_{m,c}q_{m,c}, \quad Q_c(0) = Q_{c0}, \quad 0 < t < T,$$

$$c)\ \frac{dq_{i,co,m}}{dt} = -b_{i,co,m}, \quad q_{i,co,m}(0) = 0, \quad 0 < t < t_0,$$

$$d)\ \frac{dQ_m}{dt} = -a_m(Q_m - Q_m^0) - a_{c,m}(Q_c - Q_c^0) - a'_{co,m}q_{i,co,m}, \quad Q_m(0) = Q_{m0}, \quad 0 < t < T.$$

The first equation describes turning on the process of flow of qi from elsewhere into the child channel by "supplementing" (applying a positive stimulus $b_{m,c}$ to) the "mother" point LV-8 located on the vacuous child channel, Q_c, during the time interval $[0, t_0]$ of the treatment. The second equation is the balance equation of qi in the child channel: on the left we have the rate of change of qi; the first term on the right accounts for the homeostatic effect of the channel on itself, where Q_c^0 is the "normal" level of qi; the second term accounts for the effect of the mother channel on the child channel (according to the creative cycle law); the third term accounts for the treatment effect of supplementing the mother point LV–8 on the child channel (again, according to the creative cycle law). The third equation describes turning on the process of flow of qi from elsewhere into the mother channel by "draining" (applying a negative stimulus $b_{i,co,m}$ to) the "isophasal" point SP-3 on the spleen channel, $Q_{co,m}$. SP-3 is the earth point, and the spleen channel is an earth channel, hence SP-3 is the isophasal (denoted by the subscript i) point of the spleen channel. At the same time, the spleen channel controls the kidney (mother) channel (according to the controlling cycle law). Here we identify the spleen channel by the subscripts "co,m": "controlling mother". The fourth equation is the balance equation of qi in the mother channel: on the left we have the rate of change of qi; the first term on the right accounts for the homeostatic effect of the channel on itself, where Q_m^0 is the "normal" level of qi; the second term accounts for the effect (negative in this case) of the child channel on the mother channel (according to the creative cycle law); the third term accounts for the treatment effect (positive in this case) of draining SP-3.

The simplest way to describe the healing process would be as follows:

$$(5) \qquad \frac{d\lambda}{dt} = -\left(\frac{dQ_c}{dt} + \frac{dQ_m}{dt}\right), \quad 0 < t < T.$$

Eq. (5) says that as long as total qi in the mother and child channels increase, $\frac{d\lambda}{dt} < 0$, the size of the "health region" is increasing, see eq. (1).

3. Yin Yang Balancing Channel Therapy. We adopt the following simplified model of the process of interaction of the extraordinary vessels with the twelve channels: treatment of either of the yin pairs of extraordinary vessels, treatment of PC-6 and SP-4 in our case, redistributes the qi between the six yin channels so that the distribution is more equal or balanced. Treatment of either of the yang pairs of extraordinary vessels, treatment of TW-5 and GB-41 in our case, redistributes the qi between the six yang channels so that the distribution is more equal or balanced.

We use the notation: $Q_1, Q_4, Q_5, Q_8, Q_9, Q_{12}$, for qi in the following yin channels: lung, spleen, heart, kidney, pericardium, liver. A mathematical model for the treatment, in the case of, say, the spleen channel, is as follows (assume that $t_0 \ll T$, and that all quantities in the equations below are positive):

$$
\begin{aligned}
&a) \quad \frac{dc_{i,4}}{dt} = b_{i,4}, \quad c_{i,4}(0) = 0, \quad i = 9,1,5,12,8, \quad 0 < t < t_0, \\
&b) \quad \frac{dQ_4}{dt} = c_{9,4}(Q_9 - Q_4) + c_{1,4}(Q_1 - Q_4) + c_{5,4}(Q_5 - Q_4) \\
&\qquad + c_{12,4}(Q_{12} - Q_4) + c_{8,4}(Q_8 - Q_4), \quad 0 < t < T,
\end{aligned} \tag{6}
$$

where $b_{i,4} > 0$, $c_{i,4} \geq 0$ are the coefficients. The first set of equations describes turning on the process of balancing qi in between spleen and other five yin channels during the time interval $[0, t_0]$ of the treatment. The second equation describes the above process of balancing in the case of the spleen channel. Essentially, it says that the amount of qi Q_4 in the spleen channel changes so as to equilibrate qi between the spleen and the other 5 yin channels. For example, $Q_9 - Q_4 > 0$ would lead to increase of Q_4, and hence to the decrease of $Q_9 - Q_4$, etc.

The simplest way to describe the healing process, say, for the spleen channel, would be as follows:

$$
\frac{d\lambda}{dt} = \frac{\mathrm{Q_4 - Q_4^0}}{\left|\mathrm{Q_4 - Q_4^0}\right|} \frac{\mathrm{dQ_4}}{\mathrm{dt}}, \quad 0 < t < T. \tag{7}
$$

Eq. (7) says that as long as qi in the spleen channel approaches its normal steady state Q_4^0, $\frac{d\lambda}{dt} < 0$, i.e. the size of the "health region" is increasing, see eq. (1).

4. Linking the qi based models with electrodermal measurements

Selecting appropriate measurement technologies at Step five to test the models we have developed can be nontrivial, since insufficient data yet exists to clearly validate the selection of any one technology, as each itself still needs to be researched further. Still, in spite of these difficulties, we can suggest a number of measurement technologies that might be useful at Step five: Electrodermal skin measurements at the acupuncture points and channels have already been used, and may prove useful; magnetic measurements of points and channels could also prove useful; certain biochemical or neurological tests may also prove useful. However, before a technology is used at Step five, and its measurements are taken as reliable indicators of the models tested from Steps one to three, it will need to be validated, and the theoretical linkages of Step four will also need to be examined.

Eqs (1)–(7) are written in terms of the quantitative variable Q, qi. However no simple relationship is presently known between qi and various material parameters of the body. However, one of us (WAT) has proposed a hypothetical model of how qi-flow in acupuncture channels creates a magnetic vector potential distribution, $\bar{A}$ along the physical locus of the channel [Tiller 1989, Tiller 1997] and at a magnitude proportional to the qi magnitude. From one of the two basic electrodynamic equations, this $\bar{A}$ generates an electric field, $\bar{E}$ given by

$$
\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t} \tag{8}
$$

oriented along the meridian (V=the electromagnetic potential distribution along the channel). This $\bar{E}$ pumps electrolyte ions along the channel towards the surface acupuncture point (A.P.) until the space charge produced by V produces a stationary condition. This leads to a surface ion concentration which increases exponentially with $\bar{E}$. Thus, the A.P. conductivity, σ_{AP}, should be given by an equation of the following form

$$\sigma_{AP} = \sigma^0_{AP}\left(1 + e^{eE(z-z_0)/kT}\right) \equiv \sigma^0_{AP} + \beta Q, \tag{9}$$

where β is some complex parameter which may be frequency dependent.

By such reasoning, a connection has been forged between qi and the electrical impedance z_{AP} of skin at an A.P. This allows one to access the available data on skin impedance measurements as a possible vehicle for testing eqs (1)–(7), and (9) [Tiller 1989, Rosendal 1943, Voll 1975, Motoyama 1980, Tiller 1982]. Measurement of A.P. surface electrical potential and differential impedance between an A.P. and its nearby non–A.P. skin could also be utilized to provide additional information on this issue.

Past studies of human z_{AP} in the 0 to 1 MHz range revealed the presence of two electric charge transport processes acting in series [Tiller 1989]. Each of these processes has a unique relaxation time τ_i with $\tau_1 \gg \tau_2$. The τ_1 process involves, among other things, carrier conduction along cellular interstices in the stratum corneum ($\tau_1 \approx 10 - 60$ sec). The τ_2 process involves charge transport across epidermis/dermis basal membrane ($\tau_2 \approx 5 - 30$ microsec). It is also possible that more rapid relaxation processes associated with enzymatic activity ($\tau_3 \approx 1$ nanosec) and hydrogen bonding changes ($\tau_2 \approx 1$ terasec) in the tissue fluids of the skin cells will be measured in the future.

Most of the commercial z_{AP} measurement devices in use today utilize the τ_1 process and the results are strongly dependent on (i) the use of constant voltage devices, (ii) on the magnitude of the applied voltage, (iii) the specific material used as an electrode, (iv) the area of the electrode, (v) the humidity and the temperature, and (vi) the applied pressure and the electrode contact angle with the skin [Tiller 1989, Margolin, Avants, Birch, Falk, Kleber 1997, Falk, Birch, Margolin, Avants 1997]. This technique also allows unconscious level processes acting in the operator to influence the measurement via the pressure/angle variation [Tiller 1989]. In many cases, at least one of us (WAT) feels, this leads to superior diagnostic ability on the part of the operator, because an additional information channel is being accessed. Only the Motoyama AMI device utilizes the τ_2 process and, because of the small time constant, is much less subjective then the τ_1 devices [Tiller 1989]. The B.P. (before polarization) values for A.P.'s, as measured via AMI, are thought to have a connection to qi.

Many of these device techniques utilize a small electrode ($\approx$ several mm^2 in area rather than a large electrode ≥ 1 cm^2) and thus measure z_{AP} as distinct from z_{skin} (which may be penetrated by one or more A.P.'s). Experimentally, one finds that $z_{AP} \approx (2 - 20) \times z_{skin}$, depending upon humidity, temperature, hypnogogic state, etc.

In closing this section, we point to a related research direction that may be important to include. This is that the possible correlation between z_{AP} at specific A.P. points and specific electrophysiological measurements could offer [Tiller 1989] a powerful pathway for understanding traditional concepts (e.g. electrocardiogram measurements via Holter tape recordings vs z_{AP} for points on the heart channel or respiration system electrical recordings vs z_{AP} for specific lung channel points). This type of correlation measurement may not only connect organ functions to specific A.P.'s but could also further the testing of alternative therapies and their relationship to traditional concepts.

5. Conclusions

We have proposed a five-step process for testing traditional explanatory frameworks of acupuncture that, we believe, could allow a bridge between the diverse traditional concepts and frameworks, and rigorous methods of testing them. We illustrated the diversity of traditionally based systems of acupuncture with a case study, showing three quite different approaches to assessing and treating that patient. Next we developed a general theoretical construct of health and disease that serves as a linkage between the various explanations and approaches, and general physiological functioning. Then, as illustrations of how our mathematical modelling approach can be used, we developed minimal mathematical models of each of the three diagnostic assessments and treatments, showing how each can be related back to the general health model we developed. Finally we briefly discussed measurement technologies, especially electrodermal measurements, focussing on theoretical considerations and methodological issues involved in the use of such measurement techniques.

We emphasize that the main thrust of this paper is to introduce a modeling approach which, as the authors believe, provides a precise language and framework for initializing basic research in TBSAs: beginning to formulate testable hypotheses and design experiments to test them. Our approach is a western scientific approach for dealing with a very diverse and poorly understood phenomena, for which no coherent framework has yet been established: qi, subtle energies, traditional acupuncture concepts, and their hypothesized holistic properties. We believe that by pushing our systematic approach to its limit (if such a limit exists), will allow us to uncover what lies beyond this limit, if anything.

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