# A dynamical systems approach to modeling meridians and Ki.

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*Abstract: We develop dynamical systems models of some concepts of classical acupuncture. We view the traditional acupuncture concepts as descriptions of a high order regulating mechanism in the human organism. Our approach is to translate these concepts into their minimal realizations as mathematical laws directly, without an attempt to reduce them to western concepts. This approach is in the spirit of the development of dynamical systems models of complex neural phenomena. This work is a step in the direction of developing models that can be experimentally tested (i.e. using electrodermal diagnostic instruments such as the Motoyama's AMI), analyzed mathematically and simulated on computers.*

*These models we develop here are designed to account for some clinical data such as Manaka's results on the application of low intensity stimuli to acupoints on a meridian and Motoyama's results on the propagation of electrical impulses along meridians. To describe a single meridian we use an electrical circuit model similar to the synapse membrane with two ionic channels and to negative resistance circuits studied by Omura. We also develop a mathematical model in the form of a linear five dimensional dynamical system of the so called "five phase" ("five element") laws such as "creative" cycle, "controlling" cycle, etc., in the case of a single meridian. We connect this model with the membrane type model mentioned above by assuming a simple mass action law (which says that the closed pores in a channel open at a rate proportional to their number and to the input signal), for the dependence of the conductances in the ionic channels on the input signals. This combined model is used to describe the development of a "disease" and its treatment according to the "five phase" theory. Here we interpret the "disease" as a blockage in a meridian, while the treatment initiates the unblocking process.*

## 1. Relevance of a nerve conduction type equation to modeling of the flow of Ki in a single meridian.

The analysis of the clinical data and the physical mechanisms of the propagation of electrical signals associated with the flow of Ki in meridians in Motoyama [Mo1, Mo2] and in Tiller [T1, T2] seem to indicate that the system of the Fitz Hugh-Nagumo (FHN) type describing nerve impulse propagation (see e.g. Rinzel [R] and Omura [O1]) is, probably, the simplest model to account for the observed phenomena. The equations are formulated for the voltage $V$ (deviation from the rest) across the meridian (membrane) which responds to applied stimulating current $I_{app}$ (and initial value $V(0)$) and to changes in meridian (membrane) permeability to various ion species (we assume, for simplicity, that only one ion species is present). These permeability changes also usually depend on $V$ and result in the generation of the electrical impulses.

**(i) Spatially lumped meridian model** (an isolated membrane patch). Here we neglect spatial differences in $V$ and in the meridian (membrane) properties so $V \equiv V(t)$ , i.e. the voltage depends only on time $t$. Meridian (membrane) current $I_{app}(t)$ has a capacitance component and an ionic one:

$$I_{app}(t) = C\frac{dv}{dt} + I_{ion}. \tag{1.1}$$

When this equation is non-dimensionalized and the auxiliary variable is included one obtains the FHN system of two differential equations

$$\frac{dv}{dt} = -f(v) - w - I_{app}, \tag{1.2}$$

$$\frac{dw}{dt} = \epsilon(v - \gamma w). \tag{1.3}$$

Here $v$ is the meridian (membrane) potential, $w$ is the single recovery variable (the ion-activation or deactivation), and $f(v)$ is an instantaneous current-voltage law (obtained experimentally) sometimes taken to be a cubic polynomial [R]. To understand the physical meaning of (1.3) we rewrite it in the form

$$\frac{dw}{dt} = (w_\infty(v) - w)/\tau(v).$$

This is a first order rate law with $v$—dependent "time constant" $\tau$ and the "steady state" function $w_\infty(v)$.

Qualitative properties of the system (1.2), (1.3).(see e.g. [R] ).*(i) threshold behavior*: a subthreshold stimulus, say, $I_{app}$ or the initial potential $V(0)$ results in response which immediately decreases to the resting state, however, with a suprathreshold stimulus, the response first undergoes a relatively large excursion away from the equilibrium before returning to the rest state, e.g. an action potential. *(ii) periodic solutions* for $I_{app}$ (or $v(0)$) in a certain range of values.

Assuming the qualitative behavior of solutions of (1.2), (1.3) corresponds to clinical data such as the AMI measurements, one would try next to determine experimentally the parameters in (1.2), (1.3) (Which is a nontrivial mathematical problem by itself, a so called "inverse problem") and refine the mathematical model.

**(ii) Effect of spatio-temporal distribution of the voltage and ionic permeabilities in a meridian.**

When $v$, (1.2) and (1.2) are replaced by

$$\begin{aligned} \frac{\partial v}{\partial t} &= \frac{\partial^2 v}{\partial x^2} - f(v) - w + I_{app}(x,t) \\ \frac{\partial w}{\partial t} &= \epsilon(v - \gamma w) \end{aligned} \tag{1.4}$$

Mathematical analysis and computer experiments for (1.4) reveal (see e.g. [R]) that in addition to the threshold phenomena and periodic solutions we also have *solitary waves* which are essentially spatially localized pulses moving on a constant background. Depending on values of the parameters in (1.4), these waves can have different shapes and wave speeds can be stable or unstable. Motoyama's clinical data seem to support this model. In particular, he found [Mo2] that electrical impulses propagate along meridians with the velocity in the range 4 cm/sec-50 cm/sec which is considerably smaller than the velocity of propagation of nervous impulses, which is in the range 50 cm/sec-100 m/sec. Motoyama also found that the velocity of the propagating impulses is in a continual state of change. It appears that the propagating impulses measured by Motoyama are traveling waves. And the simplest mathematical equation which has traveling waves as its solutions has

the form (1.4). Thus it would be interesting to try to determine whether the appropriate parameter changes in (1.4) can account for Motoyama's clinical results.

To conclude this section we would like to point out that the notion of *stability* might be important in interpreting the clinical data. A simple way to look at this is as follows. The Ki flow (and the corresponding electrical characteristics) in the human organism oscillate about certain physiologically relevant ("healthy" or "unhealthy") stable states. Note that in the physiological studies these stable states are usually time independent or periodic in time. So in interpreting the clinical data it is important to be aware of the distinction between the stable states (solutions) and the transient solutions. We thus feel that mathematical analysis and the computer simulation for the appropriate equations might compliment and clarify the statistical analysis of the clinical data as in [Mo3]. The work is in progress [TF] to develop a more realistic model for the flow of Ki.

## 2. A dynamical systems model for the "five phase" laws in the case of a single meridian.

Manaka's clinical results [MI2, MIB] (confirmed by our own) show that application of a small voltage to the so called "mother" and "child" points on a meridian reduces presure pain *independently* of whether the meridian was vacant or replete. Here the terms "vacancy" and "repletion" are used in the sense of traditional acupuncture. Clinical evidence of Manaka and Itaya [MI1] and others suggests the possibility that the initial electric current BP as measured by Motoyama's AMI can be interpreted as an indication of the condition (as determined by traditional acupuncture procedures) of a meridian: high BP corresponds to repletion, and low BP corresponds to vacancy.

To account for Manaka's results we introduce a nerve conduction type equation similar to that of a synapse membrane with two ionic channels:

$$c\frac{dv}{dt} = (v^+ - v)\, g^+ + (v^- - v)\, g^-, \tag{2.1}$$

where $v^+$ and $v^-$ are excitatory and inhibitory saturation points, respectively: $v^+ > v^-$; $g^+ > 0$ and $g^- > 0$ are the corresponding conductances

of two ionic channels, while $c$ is the capacitance which we can assume to be unity, for simplicity.

Thus, depending on the values of $g^+$ and $g^-$, $v$ can take any value between $v^+$ and $v^-$, which we interpret as being related to the most replete and most vacant states of the meridian, correspondingly. When the meridian becomes vacant, let us presume that the resistance of the excitatory channel becomes large (the excitatory channel closes) so that $g^+$ becomes small and $v$ is close to $v^-$: i.e. we have blockage in the excitatory channel. Similarly, if $g^-$ becomes very small, we presume that it is due to the meridian being replete and $v$ is close to $v^+$. This we interpret as blockage of the inhibitory channel (the inhibitory channel closes). For example, placing copper to the "mother" point and zinc to the "child" point reduces pressure pain on the meridian or at associated meridian points. Reversing these so that zinc is on the "mother" point and copper is on the "child" point, causes the pressure pain to return. Note that, from a simple electrochemical viewpoint, copper is electropositive relative to zinc. The observed effect of reduction of pressure pain can be explained as follows: *in the case of vacancy $g^+$ is increased which makes the meridian more replete; in the case of repletion $g^-$ is increased so that the meridian becomes more vacant* (see [FBT1, FBT2] for more details).

Omura's results [O2] seem to support our model. He has shown that low energy signals can change body musculature, but they cannot create such changes by introducing voltage to neural membranes. Low energy signals cannot create a sufficient voltage to cause nerve impulses. Our model describes such signals as affecting changes in conductances and not voltages. This is similar to the negative resistance circuit suggested by Omura [O2] as a possible mechanism of the above effects.

We next extend our model to incorporate the effects of different signals on the excitatory conductance $g^+$ (for $g^-$ the analysis is similar). Our approach here is similar to the one in Carpenter and Grossberg [CG]. We use a simple mass action law. To be specific, assume that (2.1) describes a wood (liver) meridian. Let $T_1$ be a signal leading to vacancy of liver meridian, i.e. decreasing $g^+$, say, due to emotional and dietary factors, acting for a long time (i.e. several years) from time $t_0$ to $t_1$. At time $t_1$ the diet was changed, and there was a treatment by needles, electrical polarities or light, according to the "five phase" laws, for a short time from $t_1$ to $t_2$ by a signal $T_2$. We

thus have:

$$T_1(t) = \begin{cases} 0, & t \le t_0, \\ T_1 = const, & t_0 \le t \le t_1, \\ 0, & t > t_1; \end{cases} \tag{2.2}$$

$$T_2(t) = \begin{cases} 0, & t \le t_1, \\ T_2 = const, & t_1 \le t \le t_2, \\ 0, & t > t_2. \end{cases} \tag{2.3}$$

The simplest mass action law is defined by

$$\frac{dg^+}{dt} = (H + JT_2(t))(g_0 - g^+) - JT_1(t)\, g^+, \; t \ge t_0, \tag{2.4}$$

where $g_0$ is the maximal number of opened pores (when $T_1(t) = 0$). Equation (2.4) says that closed pores, which number $g_0 - g^+$, open at a rate $H + JT_2(t)$ (when $T_2(t) = 0$ the rate is $H$); and that the signal $T_1(t)$ closes open pores, which number $g^+$, at a rate $J$. We also assume that at time $t = t_0$ all pores were open i.e. no blockages:

$$g^+(t_0) = g_0. \tag{2.5}$$

Solving (2.4), (2.5) for different time intervals gives:

$$g^+(t) = \frac{H}{H + JT_1} g_0, \quad t_0 \ll t \le t_1, \tag{2.6}$$

for time $t$ "long enough";

$$g^+(t) = g_0 - g_0 \frac{JT_1}{H + JT_1} \exp\left(-(H + JT_2)(t - t_1)\right), \; t_1 \le t \le t_2; \tag{2.7}$$

$$g^+(t) = g_0 - \left(g_0 - g^+(t_2)\right) \exp\left(-H(t - t_2)\right), \; t \ge t_2. \tag{2.8}$$

Equation (2.6) shows that the excitation channel of the liver meridian is now partially blocked (the conductance $g^+$ decreased). Equation (2.7) models the unblocking process (increase of $g^+$) due to the treatment. Equation (2.8) says that the unblocking process continues after the treatment stopped, but

with a slower rate than during the treatment. Substitution of (2.6), (2.7) and (2.8) into (2.1) and solving it will show how the voltage $v$ changes in time. Clinical experiments are required to test this model.

We next look more closely at the treatment process,e.g. how $T_2(t)$ can be obtained. Clinical experiments of Manaka and ours seem to agree with the traditional so called "five phase" theory. The following example with colors [Ma] illustrates this.

EXAMPLE. To reduce pressure pain on LI-4, a *metal* meridian point, we apply a red colored mark to LU-10, a *fire* point on a *metal* meridian. Here *fire* "controls" *metal,* red on a *metal* meridian "controls" the meridian. Applying a black colored mark to LU-10 will cause the LI-4 pressure pain to return. Here *water* "controls" *fire,* black on a *fire* point reduces *fire's* "controlling" effect on the *metal* meridian.

The simplest conceptual model of he classical "five phase" theory in the case of single meridian can be formulated as follows. Given a stimulation of one or several of the five acupoints corresponding to different phases, on the meridian under consideration, the five acupoints stimulate each other according to what the traditional theory calls "creative" and "controlling" cycles. Then the stimulation of the meridian is proportional to the resulting stimulation of the acupoint that belongs to the same phase as the meridian. The simplest mathematical interpretation of this conceptual model gives a linear five-dimensional dynamical system. We illustrate this approach in the case of our original example of the *wood* (liver) meridian.

Let $S_j(t)$, $j = 1,...,5$, (assume $j = 1$ for *water*, $j = 2$ for *wood* etc.) denote the amount of stimulation of the $j$—th point on the *wood* meridian. Suppose our treatment was to tonify the *water* point by a signal $R$. Then the simplest mathematical model describing the effects of this treatment is given by the system of five linear differential equations (see also [BF]) :

$$\begin{aligned} \frac{dS_1}{dt} &= aS_5 - bS_4 - cS_1 - dS_2 - eS_3 + R(t), \quad t \geq t_1, \\ \frac{dS_j}{dt} &= aS_{j-1} - bS_{j-2} - cS_j - dS_{j+1} - eS_{j+2}, \quad j = 2,3,4,5, \end{aligned} \tag{2.9}$$

where $R(t) = R = const > 0$, for $t_1 \leq t \leq t_2$ and $R(t) = 0$ otherwise; with the initial conditions

$$S_j(t_1) = 0, \quad j = 1,...,5. \tag{2.10}$$

Here $a, b, c, d, e > 0$. The first equation in (2.9), for example, says that the rate of change of stimulation of the *water* acupoint is proportional to the stimulation ("creative") $S_5$ of the *metal* acupoint, to negative stimulation $S_4$ (controlling) of the *earth* acupoint, to the negative stimulation $S_1$ (homeostatic, see [BF] for the discussion ) of the *water* acupoint, to the negative stimulation $S_2$ (countercreative) of the *wood* acupoint, to the negative stimulation $S_3$ (countercontrolling) of the *fire* acupoint, to the stimulation $R(t)$ from the treatment.

Solving (2.9), (2.10) we obtain, in particular, $S_2(t)$ which gives us the resulting effect of the treatment on the *wood* acupoint on the *(wood)* liver meridian. For the treatment signal $T_2(t)$ we can set $T_2(t) = S_2(t)$ or, to account for our simplified assumption on the form of $T_2(t)$ in (2.3),

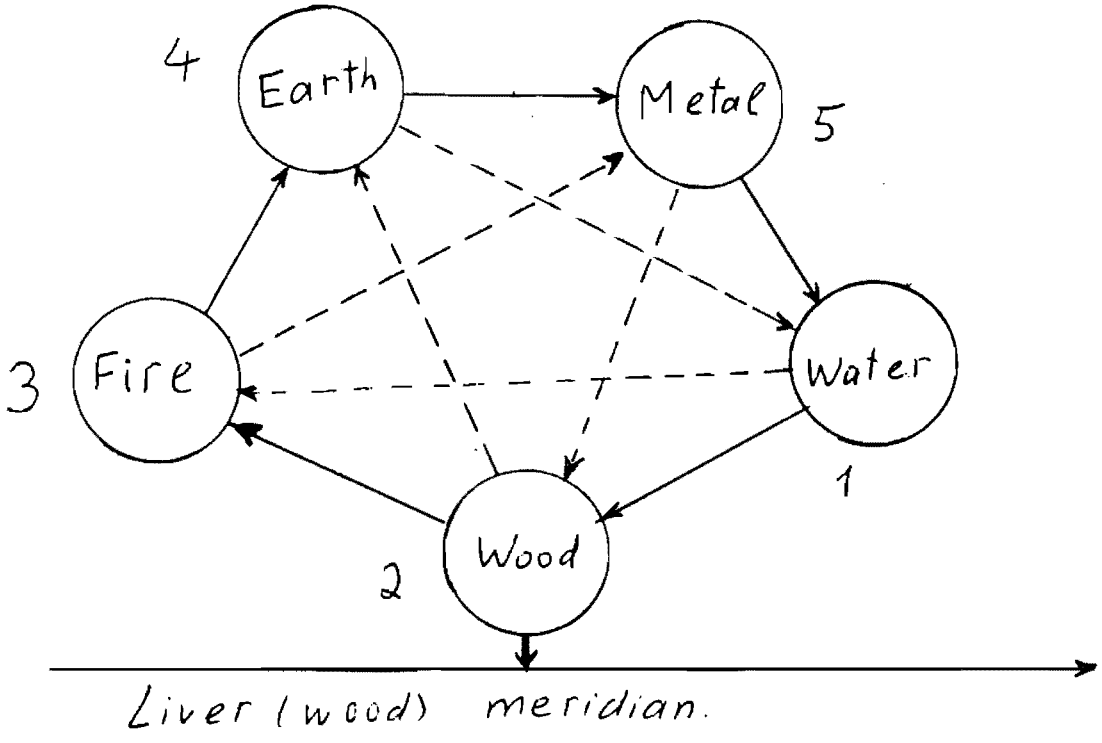
$$T_2(t) = \begin{cases} 0, & t \le t_1, \\ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} S_2(t)\, dt, & t_1 \le t \le t_2, \\ 0, & t > t_2. \end{cases} \tag{2.11}$$

We thus assume that the stimulation of the liver *(wood)* meridian equals the (average) stimulation of the *wood* acupoint.

One can see that even in the case of a single meridian we have at least 6 equations. Thus if we want to describe meridian circuits the problem becomes really complicated. A powerful code AUTO [DK] developed recently by Doedel for the analysis of the qualitative behavior of nonlinear dynamical systems, perhaps, would be appropriate for the computer simulation of meridians and meridian networks. New accurate, robust, and systematic methods for computing orbits of the dynamical systems developed by Doedel and Friedman [DF, FD] are currently being incorporated into AUTO.

**3. Discussion.** We have chosen the language of modern dynamical systems to explore mathematically some concepts and clinical data from classical acupuncture. We also indicated how to link our models with electrical measurements by electrodermal diagnostic instruments such as Motoyama's AMI. In the development of our dynamical systems models we have drawn on the models of complex neural phenomena. This is a natural first step. However the full potential of mathematical language to unify our knowledge and increase the depth of our understanding of meridians and Ki can fully manifest only in broadly designed interdisciplinary projects

combining rigorous clinical experiments with mathematical analysis and the computer simulation.

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